



The Need for Differentiating Mathematics Instruction

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Of all the challenges facing educators today, perhaps the most difficult is meeting the needs of individual learners. Within any given classroom, there will be a heterogeneous mix of ability levels—from students who perform above grade level to those who struggle to meet grade-level expectations. Yet—though differentiating reading instruction has become the norm in most classrooms—mathematics instruction rarely targets learning to the exact needs of individual students. In order to truly prepare students for success in and out of the classroom, teachers must differentiate the mathematics curriculum to meet the needs of all learners—by remediating or accelerating instruction when necessary and providing them with opportunities to learn and grow.

Differentiating Instruction with The Quantile® Framework for Mathematics

ASCD (Association for Supervision and Curriculum Development) (2009) describes differentiated instruction as a means of creating multiple paths so that students of varying abilities, interests and learning needs experience equally appropriate ways to absorb, use, develop and present concepts as a part of the daily learning process. In reading, MetaMetrics' widely adopted Lexile® Framework for Reading measures both reading ability and text difficulty on a single Lexile scale, enabling teachers to match students with materials that meet and challenge their abilities. Similarly, the Quantile Framework for Mathematics measures student mathematical ability, the curriculum and teaching materials on the same Quantile scale, and allows teachers to determine which mathematical skills and concepts a student is ready to learn and which skills and concepts will require additional instruction. Educators use this information to match students with learning resources at the right developmental level.

MetaMetrics built the Quantile scale, and its taxonomy of skills and concepts, to advance how mathematics instruction is targeted to meet the needs of individual learners. As illustrated in Figure 1, today's classrooms comprise wide ranges of student reading and mathematical abilities. For example, fifty percent of fourth graders have a Quantile measure between 455Q and 700Q. Said differently, there is a range of 245Q within the fourth grade; 25 percent of students are above 700Q and 25 percent are below 455Q (this is comparable to an interquartile Lexile range of 315L). By manipulating certain elements of instruction, educators can support the needs of all learners, regardless of where the learners are in their mathematical thinking and development.

Figure 1. Typical Grade Ranges for Students' Quantile and Lexile Measures

Grade	Quantile Measures (Interquartile ¹ Range, Mid-Year)	Lexile Measures (Interquartile ¹ Range, Mid-Year)
1	EM70Q* to 205Q	BR120L** to 295L
2	130Q to 390Q	170L to 545L
3	305Q to 555Q	415L to 760L
4	455Q to 700Q	635L to 950L
5	570Q to 820Q	770L to 1080L
6	670Q to 915Q	855L to 1165L
7	765Q to 1010Q	925L to 1235L
8	845Q to 1090Q	985L to 1295L
9 (Algebra)	915Q to 1160Q	1040L to 1350L
10 (Geometry)	975Q to 1225Q	1085L to 1400L
11&12 (Algebra II)	1030Q to 1280Q	1130L to 1440L

*Emerging Mathematician **Beginning Reader

¹ Interquartile is the range between the upper and lower quartiles (i.e. the middle 50 percent of a distribution) and is equal to the difference between the 75th and 25th percentile.

By its nature, mathematics builds upon a set of initial skills and concepts. Within the Quantile Framework, MetaMetrics has identified and empirically ordered approximately 500 mathematical skills and concepts called QSCs (Quantile Skills and Concepts). The QSCs extend over the five content strands proposed by the National Council of Teachers of Mathematics (NCTM)—geometry, measurement, numbers and operations, algebra/ patterns & functions, and data analysis & probability—and range from kindergarten skills through Algebra II topics. Each QSC has a unique knowledge cluster (Figure 2) that illustrates different types of relationships and the connections across strands that create a cohesive mathematical understanding. Knowledge clusters include prerequisite skills, supplemental skills and impending skills (what follows a particular skill). For example, in order to add two- and three-digit

Figure 2. Knowledge Cluster

IMPENDING QSC **QSC598 (90Q):** Add two- and three-digit numbers with regrouping.

PRIMARY SKILL **QSC79 (EM*):** Add two- and three-digit numbers without regrouping.

PREREQUISITE QSC **QSC41 (EM*):** Know and use addition and subtraction facts to ten.

* Emerging Mathematician (EM) represents a Quantile measure of 0Q and below.

Leveling the International Playing Field

The Quantile Framework for Mathematics can assist educators and students as they strive to develop what mathematics scholar Liping Ma (2007) describes as a “profound understanding of fundamental mathematics.” It has been well documented that Chinese students typically outperform U.S. students on international comparisons of mathematics competency, despite the fact that Chinese teachers receive far less education than their U.S. counterparts (11–12 years versus 16–18 years). Ma believes that elementary teachers need to become accomplished mathematics teachers and suggests that teaching knowledge is more common abroad because Chinese educators begin their careers with a better understanding of elementary mathematics and, more importantly, continue to improve how they teach mathematics throughout their careers. She notes that teaching conditions in the U.S., unlike those in China, have a substantial effect on the development of elementary teachers’ mathematical knowledge.

Ma recommends that changes in teacher preparation, support and mathematics education research might allow U.S. teachers to attain this “profound understanding of fundamental mathematics.”

In his book, “*Outliers: The Story of Success* (2008),” Malcolm Gladwell notes that, on average, Asian eighth-grade students work on mathematical problems much longer than U.S. students before they deem the problem too difficult to solve. Gladwell cites this persistence factor as the major difference in test scores across countries and reports that this is even reflected in the questionnaire that students complete prior to taking the Trends in International Mathematics and Science Study (TIMSS) test. TIMSS compares achievement in mathematics and science of U.S. students in grades four and eight with that of students in other countries. The rank order by questionnaire completion rate is exactly the same as the rank order of mathematics scores.

numbers without regrouping (QSC79), a student first should understand addition and subtraction facts to ten (prerequisite QSC (QSC41)). Once these skills are learned, the student then is ready to add two- and three- digit numbers with regrouping (impending QSC (QSC598)). Many students learn procedural knowledge without conceptual understanding. Prerequisite and supplemental QSCs enhance a lesson to deepen students’ understanding of mathematical concepts.

QSCs also are aligned with state standards. The Quantile Web site at www.Quantiles.com offers a number of free resources that support differentiated mathematics instruction (by state-defined accountability requirements), including more than 200 textbooks and 350 curriculum resources such as websites, games and books. These resources have been calibrated to the Quantile scale and have a Quantile measure. Educators use these Quantile measures to find mathematically appropriate material based on a specific topic and where a student is in his or her mathematical development. They also can use the online lesson planner to organize their lesson planning and include supplemental materials.

The Quantile Framework helps to validate what educators have known for some time: the range of mathematical abilities within any given classroom is significant and differentiation is neces-

sary. Just as “one size does not fit all” in reading instruction, “one size does not fit all” when teaching mathematics. The Quantile Framework captures the interconnectivity of mathematics. With this cohesive understanding of how mathematical skills and concepts fit together, educators can differentiate classroom instruction to support successful learning experiences for all students.

For more information, a list of the assessments and programs that report Quantile measures, and curriculum resources and textbooks that have Quantile measures, visit www.Quantiles.com.

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MetaMetrics, a privately held educational measurement and research organization, develops scientifically based measures of student achievement that link assessment with instruction, foster better educational practices, and improve learning by matching students with materials that meet and challenge their abilities. The organization’s team of psychometricians developed the widely adopted Lexile Framework for Reading; El Sistema Lexile para Leer, the Spanish-language version of the Lexile Framework; The Quantile Framework for Mathematics; and The Lexile Framework for Writing. In addition to licensing Lexile and Quantile measures to state departments of education, testing and instructional companies, and publishers, MetaMetrics delivers professional development, resource measurement and customized consulting services.

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