METAMETRICS RESEARCH BRIEF

Quantile Mathematical Demand for Career Accessibility

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OBJECTIVE

As evidenced by broad-based institutional initiatives (e.g., National Governors Association [NGA] Center for Best Practices, Council of Chief State School Officers [CCSSO], 2010) and United States Congressional actions (Every Student Succeeds Act [ESSA], 2015), recent educational policy discourse in the United States has promoted strategic interest in the mathematical content demands associated with college and career readiness. In previous work, Sanford-Moore et al. (2014) documented a lesson-difficulty continuum for K-12 mathematics and Williamson, Sanford-Moore and Bickel (2016) quantified mathematical content demands (and hence requisite student ability) for general college and career readiness. The purpose of this research brief is to quantify the demand of mathematical skills and concepts associated with access to specific postsecondary careers and occupations. In so doing, the student mathematics ability necessary for career *preparedness* or career *accessibility* can be inferred for specific individual careers and occupations through conjoint measurement. The current study does not focus on the mathematics demand of continuing successfully in a particular career, only the mathematics demand of career preparedness or accessibility.

Key Hypotheses: The study provides descriptive results only.

METHODS

Units of analysis:

The units of analysis for this study were mathematical skills, concepts or applications that were (a) classified as Quantile skills or concepts (QSCs) in The Quantile® Framework for Mathematics (see **Measures,** below) and furthermore (b) could be associated logically with entry into specific careers and/or occupations. Admissible mathematics content satisfied the following requirements for inclusion in the study.

- Mathematics skills/concepts selected for study were commonly required of individuals being trained for a specific career or in their first year of employment in the career. For example, mathematics skills/concepts might include (but are not limited to) those with the following characteristics:
    - Introduced in materials used for degree or certification programs required for career entry;
    - Commonly featured by professional organizations associated with the career;
    - Posted on websites of career-related professional organizations;
    - Contained in recruitment materials associated with the career; or,
    - Commonly used in materials, manuals, or references associated with on-the-job performance during the first year of employment in the career.
- Examples of the skills/concepts had to be readily available in print or editable electronic format.
- Examples of skills/concepts had to be classifiable as to source, purpose and/or use.

Career sample selection:

There were two primary sources for the vocational contexts examined in this study—Job Corps careers and O*NET Bright Outlook Occupations. Job Corps (Workforce Innovation and Opportunity Act [WIOA], 2014) is a residential education and job-training program for young adults ages 16-24 and is administered by the United States Department of Labor. The Job Corps program identifies careers by a Training and Achievement Record (TAR) Code and documents the mathematics skills and concepts associated with each career. O*NET is an online environment for vocational exploration (http://www.onetonline.org/find/career). The National Center for O*NET Development designates Bright Outlook Occupations using data collected by the U.S. Bureau of Labor Statistics (BLS). An occupation is called Bright Outlook if it (a) exhibits rapid growth, (b) has large numbers of job openings or (c) is a new and emerging occupation. O*NET identifies occupations with a Standard Occupational Classification (SOC) code.

Focal occupations for this study were selected in collaboration with Job Corps leadership and in the context of ongoing postsecondary career preparedness research conducted by MetaMetrics. Several attributes were considered favorable for selection.

- Careers that were of highest interest to Job Corps
- Bright Outlook Occupations identified by O*NET
- Mathematics content containing calibrated Quantile skills and concepts
- Careers/Occupations previously examined with regard to text demands (e.g., Williamson & Baker, 2013)

Because The Quantile Framework for Mathematics currently does not designate or quantify QSCs associated with higher mathematics content (e.g., calculus and beyond), the study necessarily focused on careers that draw on the content typical of high school mathematics curricula or college curricula through precalculus. Consequently, careers that rely predominantly on knowledge of calculus or higher level mathematics concepts were excluded from this study. Conley (2010) pointed out that “a thorough understanding of the basic concepts, principles, and techniques of algebra” (p.37) is most important for success in higher mathematics content, thus giving credence to the relevance of the

Quantile Framework for characterizing the mathematics demand at the interface between high school mathematics and postsecondary mathematics beyond precalculus.

Some careers/occupations selected for the study belonged to both O*NET and Job Corps databases. Distinguishing such careers/occupations was facilitated by a crosswalk between Job Corps TAR Codes and O*NET SOC Codes. This research brief summarizes the mathematics demand for 103 careers/occupations analyzed as of October 2017.

Procedure:
Mathematics content was collected for selected careers/occupations according to the data collection protocols described above. Subject matter experts analyzed each content sample to identify the QSCs associated with the material. The Quantile Framework for Mathematics provided the Quantile® measure associated with each QSC. The mathematical demand of a career or occupation was then represented by a statistical summary (i.e., 25th percentile, median and 75th percentile) for its distribution of QSC measures.

In some cases, an individual QSC occurred more than once in the content sample for a specific career/occupation. The unique, non-duplicated QSCs constituted the analysis sample for each individual career/occupation. Unique, non-duplicated QSCs were used to represent a career/occupation because they best represent what happens in mathematics books on which the Quantile lesson continuum was based—that is, in most mathematics lessons the QSC of the lesson is unique and not repeated between lessons. We wished to closely follow this convention for characterizing careers, as we view the career space as an extension of the K-12 mathematics continuum.

The Quantile Framework was designed first for instructional purposes and uses a 50% matching convention by default for interpreting the relationship between person ability and mathematics content demand. With this convention, whenever a student's mathematics ability (as represented by his or her Quantile measure) matches the mathematics content demand (Quantile measure) of a particular mathematics skill or concept, the Quantile Framework forecasts 50% understanding for the student with respect to the particular skill/concept. When students demonstrate 50% understanding, it means they are ready for instruction on the skill/concept.

Yet, students who aspire to enter a career are expected to be ready to perform (as opposed to being ready for instruction). Consequently, for this study, we moderated the matching convention of the Quantile Framework so that Quantile measures related to career accessibility are based on a 75% matching convention. Thus, when the difficulty of a QSC is reported for mathematical skills or concepts required in a career context, the Quantile measure represents the mathematics ability a person must possess to have 75% understanding. All career Quantile measures reported in this research brief have been moderated accordingly.

Measures:
The primary measure used for the analysis is the Quantile measure provided by The Quantile Framework for Mathematics (MetaMetrics, 2011). The Quantile Framework is a scientific approach to measuring mathematics achievement and concept/application solvability. The Quantile Framework consists of a Quantile measure and the Quantile® scale. A Quantile measure represents the difficulty of a mathematical skill, concept or application (QSC) as well as a developing mathematician's understanding of the QSCs in the areas of Geometry; Measurement; Number Sense; Numerical Operations; Algebra and Algebraic Thinking; and Data Analysis, Statistics, and Probability. Quantile measures are expressed with numerals followed by a "Q" (for example, 850Q), and are quantitatively calibrated on the Quantile scale. The Quantile Framework spans the developmental continuum from kindergarten mathematics through the content typically taught in Algebra II, Geometry, Trigonometry, and Precalculus. Quantile measures range from below 0Q (Emerging Mathematician) to above 1600Q. At present, the Quantile Framework does not designate or calibrate QSCs for higher mathematics content (e.g., calculus and beyond).

ANALYSES

Four analyses were completed using the database of 103 careers/occupations selected for the study. First, for each Job Corps career and each Bright Outlook Occupation, selected percentiles were calculated for the distribution of associated QSC difficulties. The median (50th percentile) Quantile measure provides a one-number summary of the typical mathematics demand associated with career preparedness. The 25th percentile and 75th percentile establish the boundaries of the interquartile range (IQR), which spans the middle 50% of mathematics demand for each career/occupation and provides a simple measure of the variability in mathematical demand within a career or occupation. In the second analysis, a bivariate plot of median career math demand measures versus years of education required for career entry is presented for the 103 selected careers/occupations. In the third analysis, the data were aggregated by sixteen career clusters identified by the United States Department of Education (USED). For each career cluster, the median Quantile measure and the 25th and 75th percentiles of the distribution are presented graphically. Finally in the fourth analysis, results from a parallel study of career text complexity (Williamson & Baker, 2013) are combined simultaneously with the present results to show postsecondary reading and mathematics career demands in conjunction with student reading and mathematics growth during the public school years. Results for the four analyses are presented graphically in four figures, which are located at the end of this research brief and discussed in the next section.

RESULTS & DISCUSSION

Figure 1 summarizes the mathematical demand associated with career accessibility for the 103 careers/occupations in the study. The horizontal scale indicates Quantile measures and the vertical scale represents the continuum of careers and occupations. In Figure 1, careers/occupations are displayed in order of increasing median Quantile measure. Thus, as the reader moves upward in the list of careers and occupations, the typical mathematics demand (or median QSC difficulty) increases monotonically with each successive career/occupation. It is obvious in Figure 1 that all careers and occupations in the study exhibit substantial variability in the mathematics demand (QSC difficulty) associated with career preparedness. The typical IQR (i.e., width of box plot) is approximately 400Q[1]. As a result, there is considerable overlap in the distributions of Quantile measures for many of the careers/occupations.

Eight specific careers and occupations are identified in Figure 1 to show how career accessibility may become more challenging as mathematical demand increases. From lowest median Quantile measure to highest, the exemplars are: Tax Preparers; Tellers; Financial Managers; Structural Metal Fabricators and Fitters; Optometrists; Civil Drafters; Solar Thermal Installers and Technicians; and, Diagnostic Medical Sonographers.

In Figure 2, the 103 careers and occupations are displayed in another fashion. The median mathematics content demand of each career and occupation is plotted versus the number of years of education required for entry into that career or occupation. Based on analogous research for reading (Williamson & Baker, 2013), one may expect there to be an approximate linear relationship between the mathematical demands of career accessibility and the number of years of education required to enter the career or occupation. Although the plot is visually suggestive, the statistical relationship between median Quantile measure and years of education (required for career entry) is too weak in these data to ascribe any relationship. This may be due in part to the fact that the conditional distributions in Figure 2 are truncated at the upper end due to the absence of careers that draw on higher mathematics content for the analyses in this study.

Number of years of education required for career entry was drawn primarily from O*NET. When data were not available from O*NET, our study used the BLS Occupational Outlook Handbook (OOH) (https://www.bls.gov/ooh/). There was one career we could not find in the OOH. For that career, we consulted My Next Move (https://www.mynextmove.org/).

In Figure 3, box plots summarize the distributions of QSC difficulties by USED career cluster. In this analysis, duplicate QSCs were allowed within a cluster to reflect the fact that a unique QSC could occur in multiple occupations, which in turn were classified into a single USED cluster. The horizontal axis in Figure 3 represents the Quantile scale. USED career clusters are alphabetically ordered (from top to bottom) along the vertical axis. In Figure 3, it appears that the median mathematical demand is lowest in the Hospitality and Tourism cluster (800Q) and highest in the Science, Technology, Engineering, and Mathematics cluster (1170Q). There is substantial variability of mathematical demand within clusters, with the within-cluster IQRs ranging from 360Q (Architecture and Construction) to 580Q (Human Services).

Four distinct perspectives are depicted in Figure 4: (a) student reading growth, (b) student mathematics growth, (c) career reading demands and (d) career mathematics demands. The horizontal axis in Figure 4 denotes both grade in school and years of education required for career entry. There are two vertical axes. The left vertical axis is calibrated in terms of the Lexile® scale to accommodate the graphical depiction of student reading growth and career reading demand, while the right vertical axis is calibrated in terms of the Quantile scale to accommodate the depiction of student mathematics growth and career mathematics demand. Furthermore, reading person measures and text-complexity measures are coded in blue, while mathematics person measures and mathematical demand measures are coded in red. Thus, the blue elements in Figure 4 should be referenced to the left vertical axis (Lexile scale) while the red elements in Figure 4 should be referenced to the right vertical axis (Quantile scale). These four graphical elements have the following additional meanings.

- **Student reading growth.** The blue curve in Figure 4 represents the average reading growth of 101,610 students from the end of Grade 3 to the end of Grade 11. All students were enrolled in the public schools of a southern state whose reading performance on the National Assessment of Educational Progress (NAEP) was at or above the national average during the time frame. The curve is specified by an unconditional quadratic multilevel growth model that was statistically fitted to purely longitudinal data (i.e., serial measures on the same students across occasions) for all students who progressed from grade to grade without repeating a grade and had at least one score during the study time frame (2002-2010). The longitudinal data for reading and mathematics growth were collected within the same state.
- **Student mathematics growth.** The red curve in Figure 4 represents the average mathematics growth of 101,650 students from the end of Grade 3 to the end of Grade 11. All students were enrolled in the public schools of a southern state whose mathematics performance on the National Assessment of Educational Progress (NAEP) was above the national average during the time frame. The curve is specified by an unconditional quadratic multilevel growth model that was statistically fitted to purely longitudinal data (i.e., serial measures on the same students across occasions) for all students who progressed from grade to grade without repeating a grade and had at least one score during the study time frame (2002-2010). The longitudinal data for reading and mathematics growth were collected within the same state.
- **Reading career demand.** Among the 101 careers and occupations that comprised the sample for this study, there were 77 careers/occupations for which both reading career demands and mathematics career demands were quantified. The blue diamonds in Figure 4 represent the median reading career demands for those 77 careers/occupations.
- **Mathematics career demand.** The red circles in Figure 4 represent the median mathematics career demands for the same 77 careers/occupations for which reading career demands are represented.

It should be understood that the Lexile scale and the Quantile scale are independent scales and are not psychometrically linked. Thus, reading growth cannot be directly compared to mathematics growth because the Lexile scale unit and the Quantile scale unit are not equated. So, for

[1] For comparison, consider that the average mathematics growth of students from the end of Grade 4 to the end of Grade 9 is 399Q (Williamson, 2016).

example, we cannot conclude from Figure 4 that reading performance is higher than mathematics performance simply because the reading growth curve lies higher on the Lexile scale than the mathematics growth curve lies on the Quantile scale. Neither can we infer that the reading demand of careers and occupations is higher than the mathematics demand because the blue diamonds in Figure 4 lie mostly above the red circles.

However we can draw conclusions from Figure 4 regarding the relationship between reading elements (reading growth vs. career reading demands); similarly Figure 4 supports inferences regarding the relationship between mathematics elements (mathematics growth vs. career mathematics demands). And, we can observe in Figure 4 that the correspondence between growth and career demand is somewhat different for reading than it is for mathematics. In particular, we note that the reading growth curve seems to align well with the median reading demands of these 77 careers/occupations because the curve rises to a point that appears to align roughly with the middle of the cloud of points (diamonds) representing the median reading demands for those 77 careers/occupations. On the other hand, it appears that the mathematics growth curve represents a trajectory that will surpass the mathematics demands of most of the 77 careers/occupations. There is a plausible reason that this inference is correct—namely, the fact that the study focused on more easily accessible careers and excluded careers whose mathematics demand derived predominantly from higher mathematics content (e.g., calculus). As the Quantile Framework is extended to accommodate higher mathematics content and QSCs are identified for calculus and higher level postsecondary mathematics courses, we should be able to include careers with higher mathematics demands, which in turn will pose higher challenge for students graduating from the public schools. At the very least, we may say that the reading and mathematics growth in this particular state seem to be well-aligned with the reading demands and the mathematics demands associated with career accessibility. Such an inference is a key advantage of using the conjoint measurement scales featured in the Lexile and Quantile Frameworks.

Limitations of the research:

As with nearly all research there are some limitations of this study that should be acknowledged. These relate to sample design, representation of careers in the sample, representation of the mathematics skills and concepts in the careers and occupations, the substantive scope of the study and the contemporaneous nature of the content included for analysis. All of these ultimately have some unknown impact on the generalizability of the results reported here.

- **Sample design.** The study employed a three-stage hierarchically nested convenience sample with QSCs nested within careers and occupations, which in turn were nested within sixteen official USED career clusters. However, a given QSC could occur in multiple careers/occupations; and, a specific career/occupation could appear in multiple USED clusters. The target population of careers originally included Bright Occupations identified by O*NET as of December 2012 and occupations identified by Job Corps. However, only a portion of the careers/occupations in these two sources have been analyzed to date. The choice of which careers/occupations to examine first was based largely on availability of materials. Thus, the sample was not a probability sample and traditional sampling theory cannot be used to calculate precision, construct confidence intervals or infer statistical significance of the results.
- **Representation of careers/occupations and QSCs.** The careers and occupations included in this study were drawn selectively from O*NET and Job Corps. So it is unlikely that the particular careers and occupations included in this study are representative of the entire universe of workplace careers and occupations. For example, the Bright Outlook occupations identified in December 2012 constituted only about a third of the entire O*NET occupational database. Further, the Bright Outlook occupations were by definition different from other occupations in the database as they were characterized by fast growth or large numbers of openings, or by being new, emerging occupations. Also, the careers/occupations selected for this study are not necessarily distributed proportionally with respect to the USED career clusters (e.g., currently there is very little representation in the Hospitality and Tourism cluster compared to other clusters). Consequently, inferences regarding the career mathematics demands of USED career clusters should be viewed with some caution.
- **Substantive scope.** As mentioned at the beginning, this study focuses on career preparedness or career accessibility, not sustained success within a career. So when we refer to the mathematics demand of a career or occupation, we mean the demand associated with *entry* into the career or occupation and not the demand experienced once on the job for an extended period of time (e.g., more than a year). In addition, careers and occupations predominantly requiring higher mathematics content were not included in the study because The Quantile Framework for Mathematics at present does not extend to QSCs that characterize higher mathematics content. As a result, some of the observed distributions of career mathematics demands are likely attenuated to an unknown extent.
- **Database evolution.** For this research, we used nationally respected databases to identify careers and occupations containing mathematics content demands. However, these databases are dynamic. Already O*NET has identified additional Bright Outlook Occupations and de-identified others. In addition, data elements such as years of required education are periodically updated as new information becomes available. Consequently, the results reported in this research must be regarded as provisional and subject to revision in the future. We anticipate that our Quantile Career Database will evolve in two substantively important ways: (a) to incorporate new careers and occupations and (b) to expand the content collections used to identify relevant QSCs.

Significance of the research:

Having identified some limitations of the research and called attention to cautions that are warranted for interpretation, we also feel obliged to point out advantages of the approach we used in this research and the benefits it could bring to educational and career planning applications.

- **Conjoint measurement.** The Quantile Framework for Mathematics was designed to provide conjoint measurement of an individual's mathematical ability and the demand of mathematical skills or concepts. It is thus possible to place both the person and the content demand on a common quantitative scale. This facilitates instructional applications by allowing teachers to match individuals with content of an appropriate challenge level for optimal instruction. It can similarly allow students, parents, and educators to judge a student's readiness to perform on skills and concepts required at job entry. Although mathematical ability may not be *sufficient* for career accessibility, it is unquestionably a *necessary* requirement for careers or occupations that demand competence with mathematics content.

- **Establishing a baseline.** In this research brief we have presented a first attempt to systematically quantify the mathematics content demands of specific careers and occupations. The results encompass (a) analyses of the mathematics content of well-defined sources as well as (b) quantification of the demand of rigorously-classified mathematical skills and concepts. As with any first attempt, there is room for future improvement. Yet this should not diminish the importance of establishing a baseline view of mathematics career demands which may inform both public instruction and personal career planning efforts going forward.

REFERENCES

Conley, D. T. (2010). *College and career ready: Helping all students succeed beyond high school.* San Francisco, CA: Jossey-Bass.

Every Student Succeeds Act (ESSA), Pub. L. No. 114–95, § 129 Stat. 1802 (2015).

National Governors Association Center for Best Practices, Council of Chief State School Officers. (2010). *Common Core State Standards for Mathematics*. Washington, DC: Author.

MetaMetrics. (2011). *The Quantile Framework® for Mathematics Technical Guide.* Durham, NC: Author.

Sanford-Moore, E. E., Williamson, G. L., Bickel, L., Koons, H., Baker, R. F. , & Price, R. (2014). *A quantitative task continuum for K-12 mathematics*. (MetaMetrics Research Brief). Durham, NC: MetaMetrics.

Williamson, G. L. (2016). Novel interpretations of academic growth. *Journal of Applied Educational and Policy Research, 2*(2), 15-35.

Williamson, G. L., & Baker, R. F. (2013). *Enriching the Concept of Career Preparedness by Examining Text Complexity Associated with Bright Outlook Occupations* (MetaMetrics Research Brief). Durham, NC: MetaMetrics.

Williamson, G. L., Sanford-Moore, E., & Bickel, L. (2016). *The Quantile® Framework for Mathematics Quantifies the Mathematics Ability Needed for College and Career Readiness* (MetaMetrics Research Brief). Durham, NC: MetaMetrics.

Workforce Innovation and Opportunity Act (WIOA), Pub. L. No. 113-128, § 128 Stat. 1425 (2014).

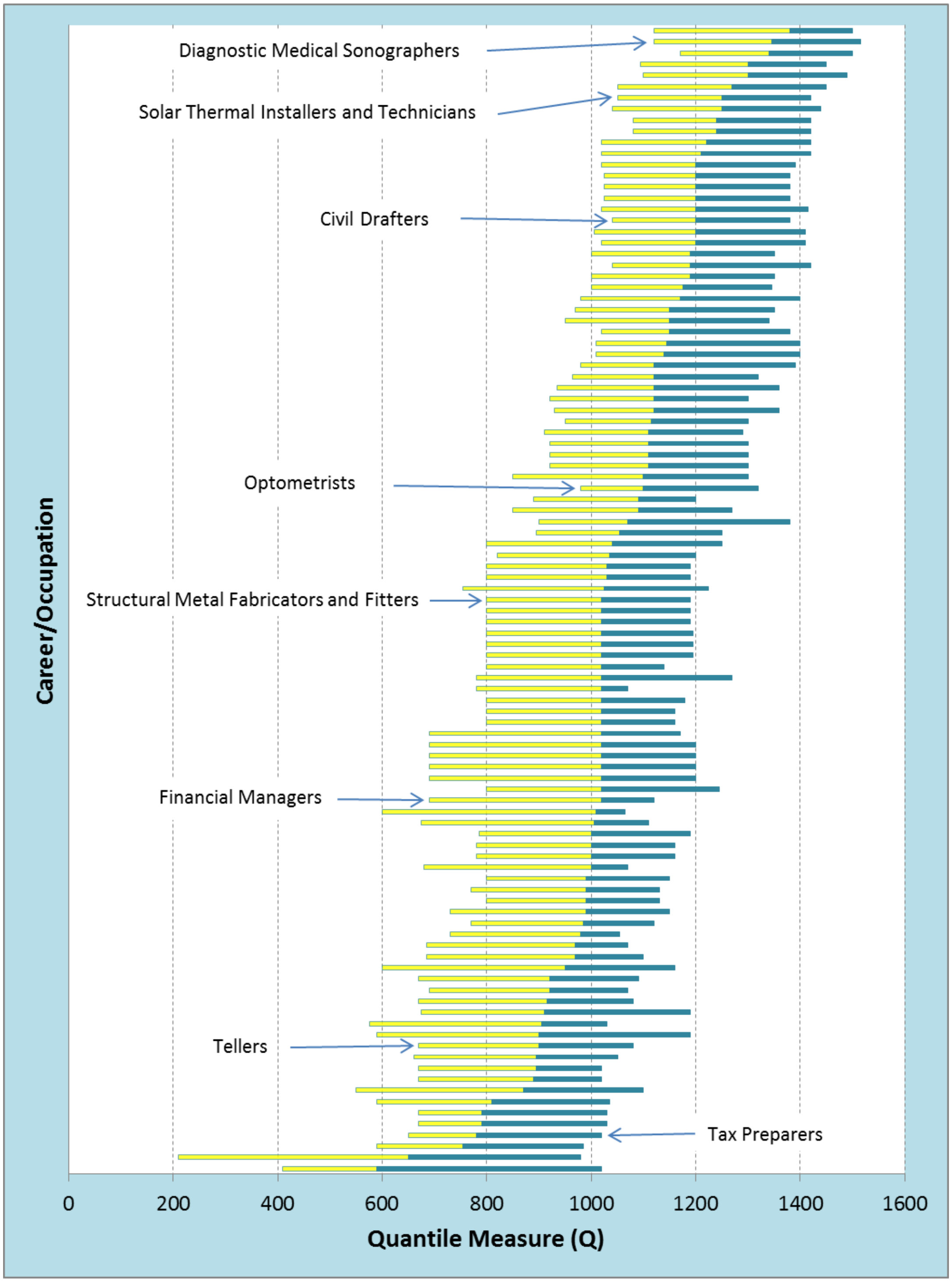

Figure 1. Mathematical content demands (difficulty of mathematical skills and concepts as measured on the Quantile scale) for 103 careers and occupations: Medians and interquartile boundaries.

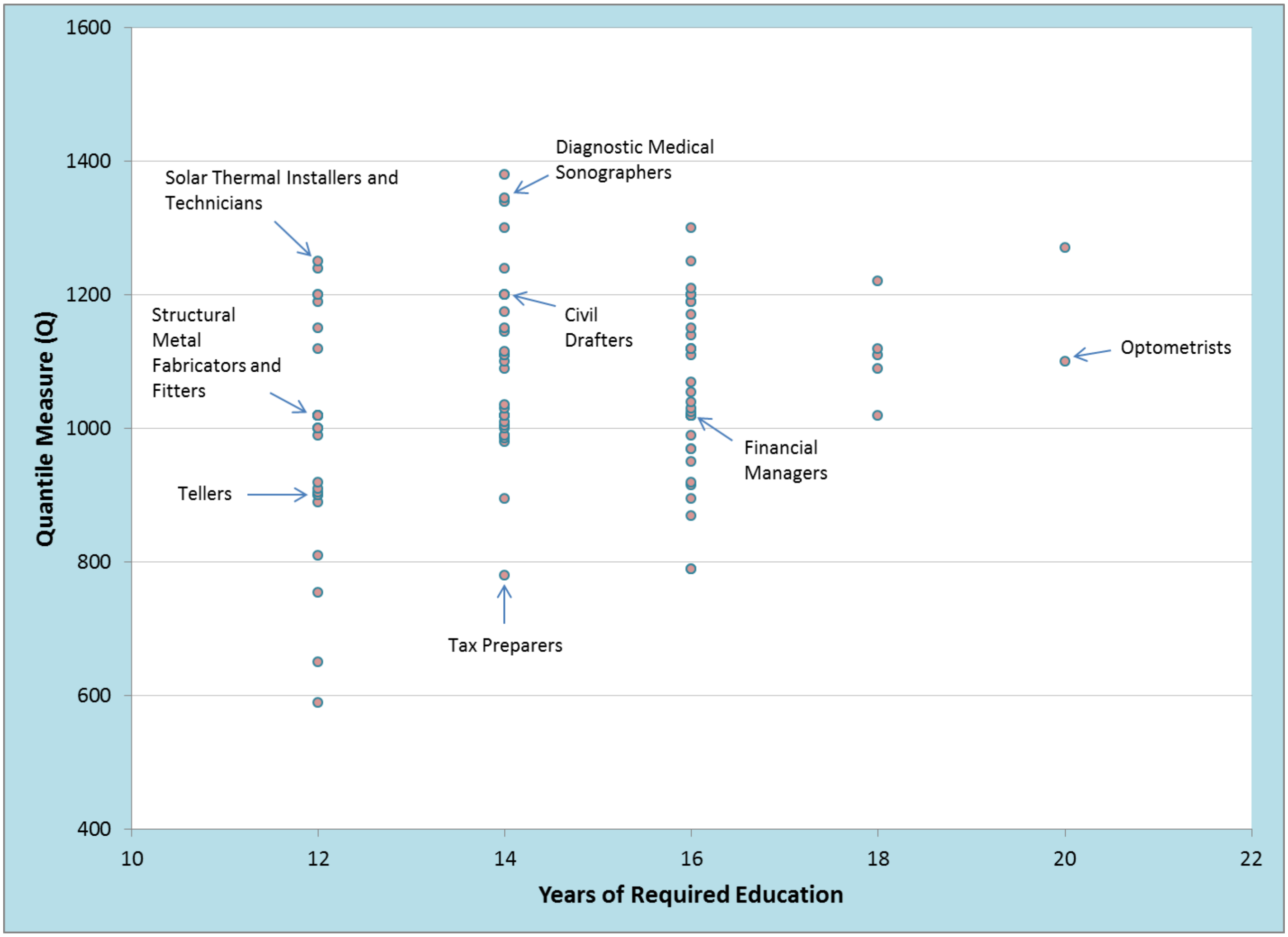

Figure 2. Median mathematics content demand (difficulty of mathematical skills and concepts as measured on the Quantile scale) versus years of education required to enter 103 careers and occupations.

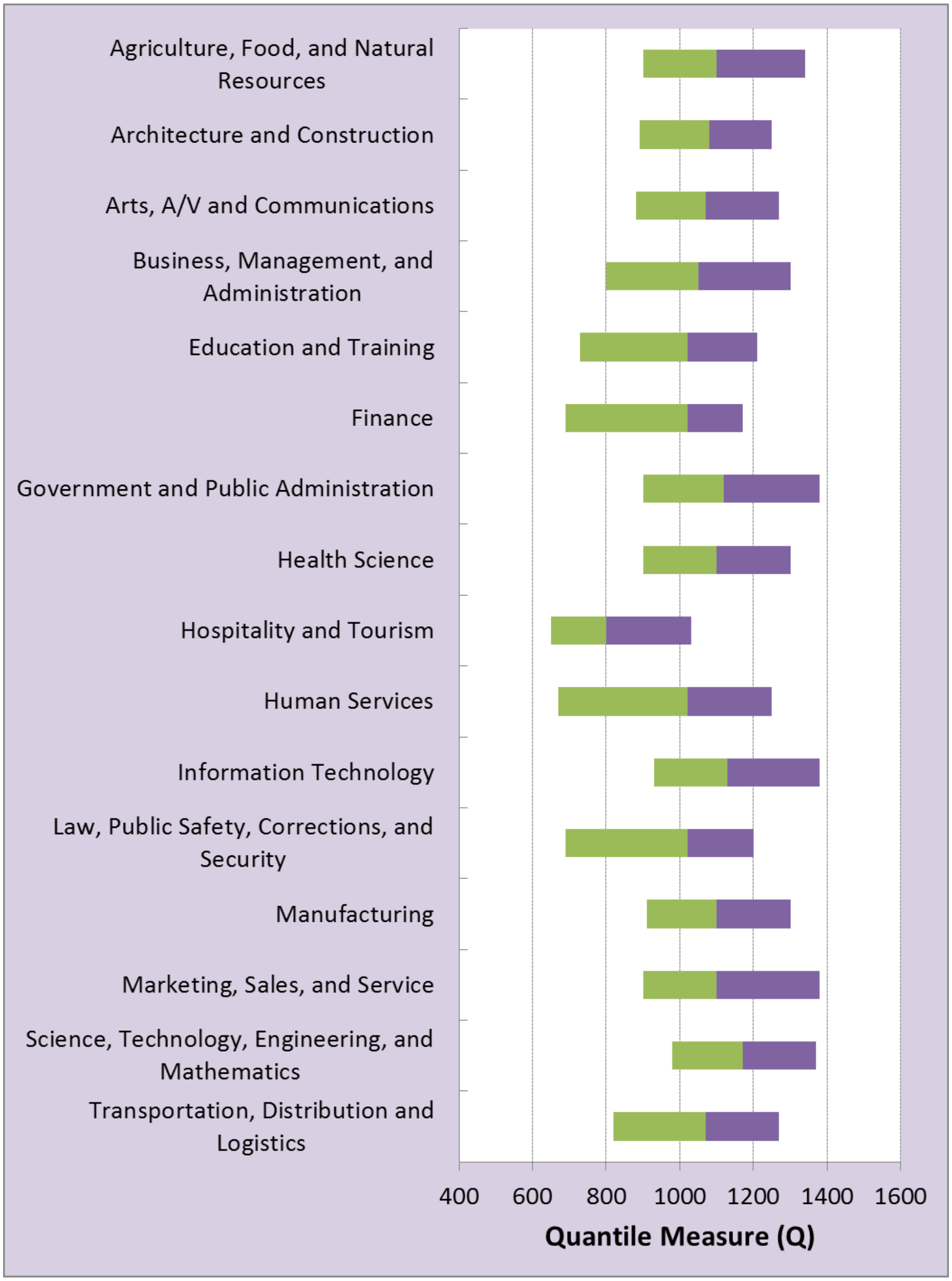

Figure 3. Mathematics content demand (difficulty of mathematical skills and concepts as measured on the Quantile scale) for 16 career clusters designated by the United States Department of Education: Medians and interquartile boundaries.

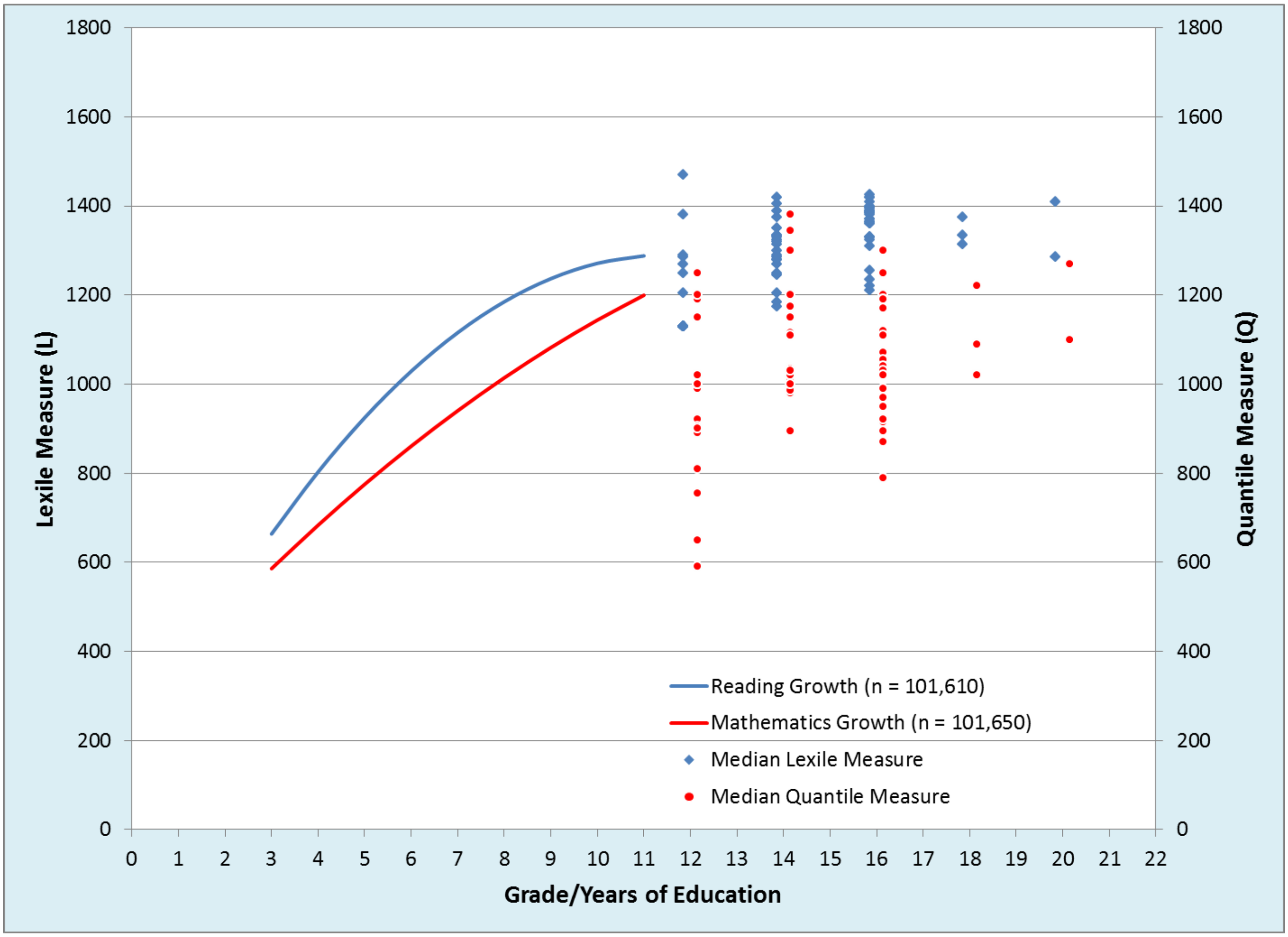

Figure 4. Average student reading and mathematics growth curves in relation to median reading demand (complexity of text as measured on the Lexile scale) and mathematics demand (difficulty of mathematical skills and concepts as measured on the Quantile scale) for 77 careers and occupations.